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DISCRETE DIRAC SYSTEM: RECTANGULAR WEYL FUNCTIONS, DIRECT AND INVERSE PROBLEMS

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Abstract. A transfer matrix function representation of the fundamental solution of the general-type discrete Dirac system, corresponding to rectangular Schur coefficients and Weyl functions, is obtained. Connections with Szegö recurrence, Schur coefficients and structured matrices are treated. A Borg-Marchenko-type uniqueness theorem is derived. Inverse problems on the interval and semi-axis are solved.

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