

COMMUTING TRACES AND LIE ISOMORPHISMS ON GENERALIZED MATRIX ALGEBRAS

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Abstract. Let \mathcal{G} be a generalized matrix algebra over a commutative ring \mathcal{R} , $q: \mathcal{G} \times \mathcal{G} \rightarrow \mathcal{G}$ be an \mathcal{R} -bilinear mapping and $\mathfrak{T}_q: \mathcal{G} \rightarrow \mathcal{G}$ be a trace of q . We describe the form of \mathfrak{T}_q satisfying the condition $\mathfrak{T}_q(G)G = G\mathfrak{T}_q(G)$ for all $G \in \mathcal{G}$. The question of when \mathfrak{T}_q has the proper form is considered. Using the aforementioned trace function, we establish sufficient conditions for each Lie isomorphism of \mathcal{G} to be almost standard. As applications we characterize Lie isomorphisms of full matrix algebras, of triangular algebras and of certain unital algebras with nontrivial idempotents. Some further research topics related to current work are proposed at the end of this article.

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