ON FUNCTIONAL EQUATIONS RELATED TO
DERIVATIONS AND BICIRCULAR PROJECTIONS

NEJC ŠIROVNIK AND JOSO VUKMAN

Abstract. In this paper we investigate some functional equations on standard operator algebras and semiprime rings. We prove, for example, the following result, which is related to a classical result of Chernoff. Let X be a real or complex Banach space, let $\mathscr{L}(X)$ be the algebra of all bounded linear operators on X and let $\mathscr{A}(X) \subset \mathscr{L}(X)$ be a standard operator algebra. Suppose there exists a linear mapping $D: \mathscr{A}(X) \to \mathscr{L}(X)$ satisfying the relation $D(A^n) = D(A)A^{n-1} + AD(A^{n-2})A + A^{n-1}D(A)$ for all $A \in \mathscr{A}(X)$, where $n > 2$ is some fixed integer. In this case D is of the form $D(A) = [A, B]$ for all $A \in \mathscr{A}(X)$ and some fixed $B \in \mathscr{L}(X)$. Some functional equations related to bicircular projections are also investigated.

REFERENCES

[1] K. I. BEIDAR, M. BREŠAR, M. A. CHEBOTAR, W. S. MARTINDALE 3RD, *On Herstein's Lie map Conjectures II*, J. Algebra 238 (2001), 239–264.

[2] F. BOTELHO, J. JAMISON, *Generalized bicircular projections on minimal ideals of operators*, Proc. Amer. Math. Soc. 136 (2008), 1397–1402.

[3] F. BOTELHO, J. JAMISON, *Generalized bicircular projections on spaces of analytic functions*, Acta Sci. Math. 75 (2009), 103–112.

[4] M. BREŠAR, J. VUKMAN, *Jordan derivations on prime rings*, Bull. Austral. Math. Soc. Vol. 37 (1988), 321–322.

[5] M. BREŠAR, *Jordan derivations on semiprime rings*, Proc. Amer. Math. Soc. 104 (1988), 1003–1006.

[6] M. BREŠAR, *Jordan mappings of semiprime rings*, J. Algebra 127 (1989), 218–228.

[7] P. R. CHERNOFF, *Representations, automorphisms and derivations of some Operator Algebras*, J. Funct. Anal. 2 (1973), 275–289.

[8] J. CUSACK, *Jordan derivations on rings*, Proc. Amer. Math. Soc. 53 (1975), 321–324.

[9] A. FOŠNER, J. VUKMAN, *Some functional equations on standard operator algebras*, Acta Math. Hungar. 118, (2008), 299–306.

[10] A. FOŠNER, D. ILIŠEVIĆ, *Generalized bicircular projections via rank preserving maps on the spaces of symmetric and antisymmetric operators*, Operators and Matrices 5 (2011), 239–260.

[11] M. FOŠNER, D. ILIŠEVIĆ, *On a class of projections on * -rings*, Commun. Algebra 33 (2005), 3293–3310.

[12] M. FOŠNER, D. ILIŠEVIĆ, CHI-K. LI, *G-invariant norms and bicircular projections*, Linear Algebra Appl. 420 (2007), 596–608.

[13] M. FOŠNER, J. VUKMAN, *On some equations in prime rings*, Monatsh. Math. Vol. 152 (2007), 135–150.

[14] M. FOŠNER, J. VUKMAN, *On some functional equations in prime rings*, Commun. Algebra Vol. 39 (2011), 2647–2658.

[15] I. N. HERSTEIN, *Jordan derivations of prime rings*, Proc. Amer. Math. Soc. 8 (1957), 1104–1119.

[16] D. ILIŠEVIĆ, *Generalized bicircular projections via the operator equation* $\alpha X^2 + AY^2 + \beta XAY + A = 0$, Linear Algebra Appl. 429 (2008), 2025–2029.

[17] D. ILIŠEVIĆ, *Generalized bicircular projections on JB^*-triples*, Linear Algebra Appl. 432 (2010), 1267–1276.

[18] J. JAMISON, *Generalized bicircular projections on spaces of operator an JB^*-triples*, Rocky Mountain. J. Math. 41 (2011), 1241–1245.

[19] I. KOSI-ULBL, J. VUKMAN, *On some functional equations on standard operator algebras*, Glasnik Mat. 44 (2009), 447–455.

[20] P.-K. LIN, *Generalized bicircular projections*, J. Math. Anal. Appl. 340 (2008), 1–4.

[21] C.-K. LIU, W.-K. SHIUE, *Generalized Jordan triple (θ, ϕ)-derivations on semiprime rings*, Taiwan. J. Math. 11 (2007), 1397–1406.

[22] A. M. SINCLAIR, *Automatic continuity of linear operators*, London Math. Soc. Lecture Note Ser. 21, Cambridge University Press, Cambridge, London, New York and Melbourne (1976).

[23] L. L. STACHÓ, B. ZALAR, *Bicircular projections on some matrix and operator spaces*, Linear Algebra Appl. 384 (2004), 9–20.

[24] L. L. STACHÓ, B. ZALAR, *Bicircular projections and characterization of Hilbert spaces*, Proc. Amer. Math. Soc. 132 (2004), 3019–3025.

[25] P. ŠEMRL, *Ring derivations on standard operator algebras*, J. Funct. Anal. Vol. 112 (1993), 318–324.

[26] N. ŠIROVNIK, J. VUKMAN, *On certain functional equation related to a class of generalized inner derivations*, Operators and Matrices, to appear.

[27] J. VUKMAN, *On automorphisms and derivations of operator algebras*, Glasnik Mat. Vol. 19 (1984), 135–138.

[28] J. VUKMAN, *On functional equations related to bicircular projections*, Glasnik. Mat. Vol. 41, (2006), 51–55.

[29] J. VUKMAN, *On derivations of standard operator algebras and semisimple H^*-algebras*, Studia Sci. Math. Hungar. 44 (2007), 57–63.