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NORMS OF POSITIVE DEFINITE TOEPLITZ MATRICES AND DETECTION OF ALMOST PERIODIC COMPONENTS IN RANDOM SIGNALS

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Abstract. For positive definite Toeplitz matrices $Q_N = (b(j-k))_{j,k=0}^{N-1}$ generated by trigonometric moments $b(j)$ of a non-negative measure $d\sigma(\theta), \theta \in [-\pi, \pi]$, we note that the Hilbert-Schmidt norm $\|Q_N\|_2$ and the maximal eigenvalue $\lambda_m(N)$ satisfy the following relations

$$\lim_{N\to\infty} \frac{1}{N^2}\|Q_N\|_2^2 = \sum_{\alpha} \mathfrak{m}_\alpha^2, \quad \lim_{N\to\infty} \frac{1}{N}\lambda_m(N) = \max_{\alpha} \mathfrak{m}_\alpha,$$

where $\{\mathfrak{m}_\alpha\}$ is the set of jumps of $\sigma(\theta)$. Analogous relations hold for positive definite integral operators with difference kernels. The above relations are used in order to detect hidden almost periodic components in random signals.

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