Operators and Matrices
Volume 8, Number 4 (2014), 935–955
doi:10.7153/oam-08-52

FREDHOLMNESS AND INDEX OF SIMPLEST SINGULAR INTEGRAL OPERATORS WITH TWO SLOWLY OSCILLATING SHIFTS

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Abstract. Let α and β be orientation-preserving diffeomorphisms (shifts) of $\mathbb{R}_+ = (0,\infty)$ onto itself with the only fixed points 0 and ∞, where the derivatives α' and β' may have discontinuities of slowly oscillating type at 0 and ∞. For $p \in (1,\infty)$, we consider the weighted shift operators U_α and U_β given on the Lebesgue space $L^p(\mathbb{R}_+)$ by $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$ and $U_\beta f = (\beta')^{1/p}(f \circ \beta)$. We apply the theory of Mellin pseudodifferential operators with symbols of limited smoothness to study the simplest singular integral operators with two shifts $A_{ij} = U_\alpha^i P_+ + U_\beta^j P_-$ on the space $L^p(\mathbb{R}_+)$, where $P_\pm = (I \pm S)/2$ are operators associated to the Cauchy singular integral operator S, and $i,j \in \mathbb{Z}$. We prove that all A_{ij} are Fredholm operators on $L^p(\mathbb{R}_+)$ and have zero indices.

Mathematics subject classification (2010): Primary 45E05; Secondary 47A53, 47B35, 47G10, 47G30.
Keywords and phrases: Slowly oscillating shift, Cauchy singular integral operator, Fredholmness, index, Mellin pseudodifferential operator.

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