FREDHOLMNESS AND INDEX OF SIMPLEST SINGULAR INTEGRAL OPERATORS WITH TWO SLOWLY OSCILLATING SHIFTS

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Abstract. Let $\alpha$ and $\beta$ be orientation-preserving diffeomorphisms (shifts) of $\mathbb{R}_+ = (0, \infty)$ onto itself with the only fixed points 0 and $\infty$, where the derivatives $\alpha'$ and $\beta'$ may have discontinuities of slowly oscillating type at 0 and $\infty$. For $p \in (1, \infty)$, we consider the weighted shift operators $U_{\alpha}$ and $U_{\beta}$ given on the Lebesgue space $L^p(\mathbb{R}_+)$ by $U_{\alpha}f = (\alpha')^{1/p} (f \circ \alpha)$ and $U_{\beta}f = (\beta')^{1/p} (f \circ \beta)$. We apply the theory of Mellin pseudodifferential operators with symbols of limited smoothness to study the simplest singular integral operators with two shifts $A_{ij} = U_{\alpha}P_+ + U_{\beta}P_-$ on the space $L^p(\mathbb{R}_+)$, where $P_{\pm} = (I \pm S)/2$ are operators associated to the Cauchy singular integral operator $S$, and $i, j \in \mathbb{Z}$. We prove that all $A_{ij}$ are Fredholm operators on $L^p(\mathbb{R}_+)$ and have zero indices.


Keywords and phrases: Slowly oscillating shift, Cauchy singular integral operator, Fredholmness, index, Mellin pseudodifferential operator.

REFERENCES


