

## PROPERTIES OF COMPLEX SYMMETRIC OPERATORS

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**Abstract.** An operator  $T \in \mathcal{L}(\mathcal{H})$  is said to be complex symmetric if there exists a conjugation  $C$  on  $\mathcal{H}$  such that  $T = CT^*C$ . In this paper, we prove that every complex symmetric operator is biquasitriangular. Also, we show that if a complex symmetric operator  $T$  is weakly hypercyclic, then both  $T$  and  $T^*$  have the single-valued extension property and that if  $T$  is a complex symmetric operator which has the property  $(\delta)$ , then Weyl's theorem holds for  $f(T)$  and  $f(T)^*$  where  $f$  is any analytic function in a neighborhood of  $\sigma(T)$ . Finally, we establish equivalence relations among Weyl type theorems for complex symmetric operators.

**Mathematics subject classification (2010):** Primary 47A11, 47A53.

**Keywords and phrases:** complex symmetric operator, biquasitriangular, weakly hypercyclic, Weyl type theorem.

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