

INJECTIVITY IN THE QUANTUM SPACE FRAMEWORK

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Abstract. In this paper we investigate injectivity of quantum (or local operator) spaces in terms of their bounded parts. A multinormed W^* -algebra with its injective domain turns out to be injective if and only if its bounded part is injective in the normed sense. We prove that each locally finite domain is injective and propose an example of a non-injective domain based on affine schemes. Hamana-Ruan type formula has been obtained for quantum spaces but in a slightly different shape.

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