

2-SUMMING OPERATORS ON $l_2(\mathcal{X})$

DUMITRU POPA

Abstract. Let $\mathcal{X} = (X_n)_{n \in \mathbb{N}}$ be a sequence of Banach spaces and $l_2(\mathcal{X})$, $c_0(\mathcal{X})$ the corresponding vector valued sequence spaces. In this paper we characterize nuclear operators on $c_0(\mathcal{X})$. As an application we obtain the necessary condition for an operator on $l_2(\mathcal{X})$ to be 2-summing. In the case of multiplication operators from $l_2(\mathcal{X})$ into $l_2(\mathcal{Y})$ (respectively from $c_0(\mathcal{X})$ into $c_0(\mathcal{Y})$) we show that the sufficient condition stated by Nahoum is also necessary. We also give the necessary and sufficient conditions for a bounded linear operator from $l_2(\mathcal{H})$ into $l_2(\mathcal{K})$ to be 2-summing, where $\mathcal{H}$ and $\mathcal{K}$ are sequences of Hilbert spaces. Further we give the necessary and/or sufficient conditions that Hardy and Hilbert type operators from $l_2(\mathcal{X})$ into $l_2(Y)$ to be 2-summing.

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