SPECTRAL PROPERTIES OF NORMAL OPERATORS
HAVING SYMMETRIES ARISING FROM CONJUGATIONS

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Abstract. We study the consequences of equations such as $AB = BA$ and $AB = BA^*$ on the spectrum of B when A is a normal operator that is real or complex skew-symmetric. Our main result is a spectral pairing theorem for such operators which generalizes results about normal matrices obtained in [11]. Important tools used are the properties of conjugations and complex symmetric operators. This paper continues the study initiated in [14].

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