

A RADON–NIKODYM TYPE THEOREM FOR α –COMPLETELY POSITIVE MAPS ON GROUPS

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Abstract. We show that an operator valued α -completely positive map on a group G is given by a unitary representation of G on a Krein space which satisfies certain conditions. Moreover, two such of unitary representations, which are unitarily equivalent, define the same α -completely positive map. Also we introduce a pre-order relation on the collection of α -completely positive maps on a group and we characterize this relation in terms of the unitary representation associated to each map.

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