

## SOME INEQUALITIES FOR UNITARILY INVARIANT NORMS

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*Abstract.* In this note, we use the convexity of the function  $\varphi(v)$  to sharpen the matrix version of the Heinz means, where  $\varphi(v)$  is defined as  $\varphi(v) = \|A^v X B^{1-v} + A^{1-v} X B^v\|$  on  $[0, 1]$  for  $A, B, X \in M_n$  such that  $A$  and  $B$  are positive semidefinite, and also give a refinement of the inequality [Theorem 6, SIAM J. Matrix Anal. Appl. 20 (1998), 466–470] which is due to Zhan.

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