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DERIVABLE MAPS AND GENERALIZED DERIVATIONS

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Abstract. Let $\mathscr{A}$ be a unital algebra, $\mathscr{M}$ be an $\mathscr{A}$-bimodule, $L(\mathscr{A},\mathscr{M})$ be the set of all linear maps from $\mathscr{A}$ to $\mathscr{M}$, and $\mathscr{R}_{\mathscr{A}}$ be a relation on $\mathscr{A}$. A map $\delta \in L(\mathscr{A},\mathscr{M})$ is called *derivable on* $\mathscr{R}_{\mathscr{A}}$ if $\delta(AB) = \delta(A)B + A\delta(B)$ for all $(A,B) \in \mathscr{R}_{\mathscr{A}}$. One purpose of this paper is to propose the study of derivable maps on a new, but natural, relation $\mathscr{R}_{\mathscr{A}}$. Moreover, we give a characterization of generalized derivations on $\mathscr{M}_n(\mathbb{C})$, the $n \times n$ matrix algebra over the complex numbers; specifically, a linear map δ on $\mathscr{M}_n(\mathbb{C})$ is a generalized derivation iff there exists an $M \in \mathscr{M}_n(\mathbb{C})$ such that $\delta(AB) = \delta(A)B + A\delta(B)$, for all $A,B \in \mathscr{M}_n(\mathbb{C})$ satisfying $AMB = 0$; in this case $\delta(I) = cM$, for some $c \in \mathbb{C}$.

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