

## FACTORIZATION OF SOME TRIANGULAR MATRIX FUNCTIONS AND ITS APPLICATIONS

YU. I. KARLOVICH, J. LORETO-HERNÁNDEZ AND I. M. SPITKOVSKY

*Abstract.* We consider defined on the real line  $\mathbb{R}$  matrix functions with monomial terms of the form  $ce^{i\lambda x}$  on the main diagonal and one row, and with zero entries elsewhere. The factorability of such matrices is established and, moreover, the algorithm for their factorization is provided. In particular, formulas for the partial indices are derived, and conditions for them to all equal zero (that is, for the factorization to be canonical) are stated. These results are then used to obtain Fredholmness criteria for some convolution type equations on unions of intervals.

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