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A GENERALIZATION OF THE BROWN–PEARCY THEOREM

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Abstract. Let $\mathscr{A}$ be a unital separable simple exact C*-algebra. Suppose that either

1. $\mathscr{A}$ is purely infinite, or
2. $\mathscr{A}\otimes\mathscr{K}$ has strict comparison of positive elements and stable rank one, and $\mathscr{A}$ has unique tracial state.

Then for all $X\in\mathscr{M}(\mathscr{A}\otimes\mathscr{K})$, X is a commutator if and only if X does not have the form $\alpha 1_{\mathscr{M}(\mathscr{A}\otimes\mathscr{K})}+x$, for some $\alpha\in\mathbb{C}-\{0\}$ and for some x belonging to a proper ideal of $\mathscr{M}(\mathscr{A}\otimes\mathscr{K})$.

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