

## SIMULTANEOUS EXTENSIONS OF A FAMILY OF LINEAR OPERATORS

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*Abstract.* In this paper we study when an arbitrary family of (positive) linear operators, defined on vector subspaces in an ordered vector space  $E$  with values in a Dedekind complete ordered vector space  $F$ , has a (positive) linear simultaneous extension. Some previous results of the first author concerning the existence of a (positive) linear common extension for two (positive) linear operators are generalized in the line of an appropriate theorem of D. Maharam. The results obtained are related to the classical Mazur-Orlicz theorem and its famous consequence, the Hahn-Banach theorem. Some applications of the main results pertaining to convex analysis, functional analysis and vector measure theory are given.

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