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PSEUDOSPECTRUM AND CONDITION SPECTRUM

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Abstract. For $0 < \varepsilon < 1$, the ε-condition spectrum of an element $A \in \mathbb{C}^{N\times N}$, a generalization of eigenvalues, is denoted by $\sigma_\varepsilon(A)$, and is defined as ([7]),

$$\sigma_\varepsilon(A) := \left\{ z \in \mathbb{C} : zI - A \text{ is not invertible or } \|zI-A\|\,\|(zI-A)^{-1}\| \geqslant \frac{1}{\varepsilon} \right\}.$$

Several results on spectrum and ε-pseudospectrum are generalized to ε-condition spectrum. The ε-condition spectrum is a useful tool in the numerical solution of operator equations. In [3], the authors have given an analogue of the Spectral Mapping Theorem for condition spectrum. This paper is a continuation of the papers [5] and [3], generalizing the Spectral Mapping Theorem for eigenvalues. In this paper we are studying size of the components of condition spectrum of a matrix. The main contribution of this paper consists of asymptotic expansions of quantities which determine the size of components of condition spectral sets. A relation connecting pseudospectrum and condition spectrum of a matrix is given as set inclusions. Using this relation a weak version of component wise condition Spectral Mapping Theorem is given. Examples are given to illustrate the theory developed.

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