

## A TRANSMISSION PROBLEM FOR THE HELMHOLTZ EQUATION WITH HIGHER ORDER BOUNDARY CONDITIONS

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*Abstract.* The present paper deals with some properties for certain classes of Wiener-Hopf operators associated with a wave diffraction problem. This diffraction problem is mathematically modeled by the Helmholtz equation and higher order boundary conditions on an infinite strip. Different types of operator relations are exhibited for different kinds of operators acting between Lebesgue and Bessel potential spaces on a finite interval and on the positive half-line. In particular, the operators under study are analyzed in detail in what concerns their Fredholm property. At the end, an operator normalization procedure is applied to the critical orders of the spaces where the problem is not normally solvable.

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