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MORE ON THE MINIMUM SKEW–RANK OF GRAPHS

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Abstract. The minimum (maximum) skew-rank of a simple graph G over real field is the smallest (largest) possible rank among all skew-symmetric matrices over real field whose ij-th entry is nonzero whenever v_iv_j is an edge in G and is zero otherwise. In this paper we obtain more results about the minimum skew-rank of graphs. Further we get a lower (upper) bound for minimum (maximum) skew-rank of unicyclic graph of order n with girth k, and characterize unicyclic graphs attaining the extremal values. Moreover, we characterize the unicyclic graphs with skew-rank 4 or 6, respectively. Finally we consider the non-singularity of skew-symmetric matrices described by unicyclic graphs.

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REFERENCES

[1] C. ADIGA, R. BALAKRISHNAN, WASIN SO, *The skew-energy of a digraph*, Linear Algebra Appl., **432**: 1825–1835, 2010.
[2] F. BARIOLI, S. M. FALLAT, L. HOGBEN, *Computation of minimal rank and path cover number for graphs*, Linear Algebra Appl., **392**: 289–303, 2004.
[3] F. BARIOLI, S. M. FALLAT, D. HERSHKOWITZ, H. T. HALL, L. HOGBEN, H. VAN DER HOLST, B. SHADER, *On the minimum rank of not necessarily symmetric matrices: a preliminary study*, Electro. J. Linear Algebra, **18**: 126–145, 2009.
[4] F. BARIOLI, S. M. FALLAT, R. L. SMITH, *On acyclic and unicyclic graphs whose minimum rank equals the diameter*, Linear Algebra Appl., **429**: 1568–1578, 2008.
[5] W. BARRETT, M. KEMPTON, N. MALLOY, C. NELSON, W. SEXTON, J. SINKOVIC, *Decompositions of minimum rank matrices*, Linear Algebra Appl., **438**: 3913–3948, 2013.
[6] W. BARRETT, H. VAN DER HOLST, R. LOEWY, *Graphs whose minimal rank is two*, Electro. J. Linear Algebra, **11**: 258–280, 2004.
[7] W. BARRETT, H. VAN DER HOLST, R. LOEWY, *Graphs whose minimal rank is two: the finite fields case*, Electro. J. Linear Algebra, **14**: 32–42, 2005.
[8] W. BARRETT, J. GROUT, R. LOEWY, *The minimum rank problem over the finite field of order 2: minimum rank 3*, Linear Algebra Appl., **430**: 890–923, 2009.
[9] N. L. CHENETTE, S. V. DROMS, L. HOGBEN, R. MIKKELSON, O. PRYPOROVA, *Minimum rank of a graph over an arbitrary field*, Electro. J. Linear Algebra, **16**: 183–186, 2007.
[10] L. M. DEALBA, J. GROUT, L. HOGBEN, R. MIKKELSON, K. RASMUSSEN, *Universally optimal matrices and field independence of the minimum rank of a graph*, Electro. J. Linear Algebra, **18**: 403–419, 2009.
[11] L. M. DEALBA, *Acyclic and unicyclic graphs whose minimum skew rank is equal to the minimum skew rank of a diametrical path*, arXiv:1107.2170v1.
[12] L. M. DEALBA, E. KERZNER, S. TUCKER, *A note on the minimum skew rank of powers of graphs*, arXiv:1107.2450v1.
[13] L. DELOSS, *Results on minimum skew rank of matrices described by a graph*, MS Thesis. Iowa State University, May 2009.
[14] S. FALLAT AND L. HOGBEN, *The minimum rank of symmetric matrices described by a graph: A survey*, Linear Algebra Appl., **426/2-3**: 558–582, 2007.

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[15] S. GONG, Y. FAN, Z. YIN, *On the nullity of graphs with pendant trees*, Linear Algebra Appl., **433**: 1374–1380, 2010.
[16] S. GONG, G. XU, *The characteristic polynomial and the matching polynomial of a weighted oriented graph*, Linear Algebra Appl., **436**: 3597–3607, 2012.
[17] J. GROUT, *The minimum rank problem over finite fields*, Electro. J. Linear Algebra, **20**: 673–698, 2010.
[18] L. HOGBEN, *Minimum rank problems*, Linear Algebra Appl., **432**: 1961–1974, 2010.
[19] IMA-ISU research group on minimum rank, *Minimum rank of skew-symmetric matrices described by a graph*, Linear Algebra Appl., **432**: 2457-2472, 2010.
[20] R. MIKKELSON, *Minimum rank of graphs that allow loops*, Ph. D. Thesis, Iowa State University, 2008.
[21] Y. WANG, B. ZHOU, *A note on the minimum skew rank of a graph*, arXiv:1206.3409v1.