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# A NOTE ON 2–LOCAL REPRESENTATIONS OF C$^*$–ALGEBRAS

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*Abstract.* We survey the results on linear local and 2-local homomorphisms and zero products preserving operators between C$^*$-algebras, and we incorporate some new precise observations and results to prove that every bounded linear 2-local homomorphism between C$^*$-algebras is a homomorphism. Consequently, every linear 2-local $^*$-homomorphism between C$^*$-algebras is a $^*$-homomorphism.

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