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COMPLEX SYMMETRIC TRIANGULAR OPERATORS

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Abstract. In this paper we explore complex symmetric operators with eigenvalues. We develop new techniques to give a geometric description of certain complex symmetric triangular operators. This extends a recent result of L. Balayan and S. Garcia concerning finite-dimensional complex symmetric operators. On the other hand, using Apostol's triangular representation for Hilbert space operators, we give a description of the internal structure of complex symmetric operators.

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Paper OaM-09-22

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