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SPECTRALLY TWO–UNIFORM FRAMES FOR ERASURES

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Abstract. We continue to work on the problem of characterizing erasure-optimal frames when spectral radius is used as a measurement of the error operator. Spectrally optimal (N,n)-frames for one erasures are the ones that the minimal spectral error n/N can be achieved. This class of frames was completely characterized in [28] in terms of the connectivity property and the redundancy distributions of the involved frames. We show that the best spectral error for the two erasures is always greater than or equal to $\frac{n}{N}+(\frac{Nn-n^2}{N^2(N-1)})^{1/2}$. We characterize all the frames such that the above lower bound can be achieved. Different characterizations are also obtained for the case that when $N=n+1$ or $n+2$. We show that in these special cases, spectrally 2-erasure optimal frames are related to the n-independence property of frames.

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Paper OaM-09-23

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