

## ON THE KERNEL OF A SINGULAR INTEGRAL OPERATOR WITH NON-CARLEMAN SHIFT AND CONJUGATION

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*Abstract.* On the Hilbert space  $\tilde{L}_2(\mathbb{T})$  the singular integral operator with non-Carleman shift and conjugation  $K = P_+ + (aI + AC)P_-$  is considered, where  $P_{\pm}$  are the Cauchy projectors,  $A = \sum_{j=0}^m a_j U^j$ ,  $a, a_j$ ,  $j = \overline{1, m}$ , are continuous functions on the unit circle  $\mathbb{T}$ ,  $U$  is the shift operator and  $C$  is the operator of complex conjugation. Some estimates for the dimension of the kernel of the operator  $K$  are obtained.

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