THE GLOW OF FOURIER MATRICES: UNIVERSALITY AND FLUCTUATIONS

TEODOR BANICA

Abstract. The glow of an Hadamard matrix $H \in M_N(\mathbb{C})$ is the probability measure $\mu \in \mathcal{P}(\mathbb{C})$ describing the distribution of $\varphi(a, b) = \langle a, Hb \rangle$, where $a, b \in \mathbb{T}^N$ are random. We prove that $\varphi/N$ becomes complex Gaussian with $N \to \infty$, and that the universality holds as well at order 2. In the case of a Fourier matrix, $F_G \in M_N(\mathbb{C})$ with $|G| = N$, the universality holds up to order 4, and the fluctuations are encoded by certain subtle integrals, which appear in connection with several Hadamard-related questions. In the Walsh matrix case, $G = \mathbb{Z}_2^n$, we conjecture that the glow is polynomial in $N = 2^n$.


Keywords and phrases: Hadamard matrix, Random matrix.

REFERENCES