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THE GLOW OF FOURIER MATRICES: UNIVERSALITY AND FLUCTUATIONS

TEODOR BANICA

Abstract. The glow of an Hadamard matrix $H \in M_N(\mathbb{C})$ is the probability measure $\mu \in \mathscr{P}(\mathbb{C})$ describing the distribution of $\varphi(a,b) = \langle a, Hb \rangle$, where $a,b \in \mathbb{T}^N$ are random. We prove that φ/N becomes complex Gaussian with $N \to \infty$, and that the universality holds as well at order 2. In the case of a Fourier matrix, $F_G \in M_N(\mathbb{C})$ with $|G| = N$, the universality holds up to order 4, and the fluctuations are encoded by certain subtle integrals, which appear in connection with several Hadamard-related questions. In the Walsh matrix case, $G = \mathbb{Z}_2^n$, we conjecture that the glow is polynomial in $N = 2^n$.

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