Operators and Matrices
Volume 9, Number 3 (2015), 481–506
doi:10.7153/oam-09-30

SPECTRAL POINTS OF TYPE π_+ AND TYPE π_- OF CLOSED OPERATORS IN INDEFINITE INNER PRODUCT SPACES

FRIEDRICH PHILIPP AND CARSTEN TRUNK

Abstract. We introduce the notion of spectral points of type π_+ and type π_- of closed operators A in a Hilbert space which is equipped with an indefinite inner product. It is shown that these points are stable under compact perturbations. In the second part of the paper we assume that A is symmetric with respect to the indefinite inner product and prove that the growth of the resolvent of A is of finite order in a neighborhood of a real spectral point of type π_+ or π_- which is not in the interior of the spectrum of A. Finally, we prove that there exists a local spectral function on intervals of type π_+ or π_-.

Mathematics subject classification (2010): 47A10, 47B50, 46C20, 47A55.

Keywords and phrases: Indefinite inner product, selfadjoint operator, spectrum of positive and negative type, spectrum of type π_+ and π_-, local spectral function, perturbation theory.

REFERENCES

[1] T. YA. AZIZOV, J. BEHRNDT, P. JONAS, AND C. TRUNK, *Spectral points of definite type and type π for linear operators and relations in Krein spaces*, J. London Math. Soc. **83** (2011), 768–788.

[2] T. YA. AZIZOV, M. DENISOV, AND F. PHILIPP, *Spectral functions of products of selfadjoint operators*, Math. Nachr. **285** (2012), 1711–1728.

[3] T. YA. AZIZOV AND I. S. IOKHVIDOV, *Linear Operators in Spaces with an Indefinite Metric*, John Wiley & Sons 1989.

[4] T. YA. AZIZOV, P. JONAS, AND C. TRUNK, *Spectral points of type π_+ and π_- of self-adjoint operators in Krein spaces*, J. Funct. Anal. **226** (2005), 114–137.

[5] J. BEHRNDT AND F. PHILIPP, *Spectral analysis of singular ordinary differential operators with indefinite weights*, J. Differential Equations **248** (2010), 2015–2037.

[6] J. BEHRNDT, F. PHILIPP, AND C. TRUNK, *Properties of the spectrum of type π_+ and type π_- of self-adjoint operators in Krein spaces*, Methods Funct. Anal. Topology **12** (2006), 326–340.

[7] C. M. BENDER, D. C. BRODY, AND H. F. JONES, *Complex extension of Quantum Mechanics*, Phys. Rev. Lett. **89** (2002), 270401.

[8] C. M. BENDER, D. C. BRODY, AND H. F. JONES, *Must a Hamiltonian be Hermitian?*, Am. J. Phys. **71** (2003), 1095.

[9] J. BOGNAR, *Indefinite Inner Product Spaces*, Springer, 1974.

[10] E. CALICETI, S. GRAFFI, AND J. SJÖSTRAND, *Spectra of $\mathcal{PT}$-symmetric operators and perturbation theory*, J. Phys. A: Math. Gen. **38** (2005), 185–193.

[11] I. COLOJOARĂ AND C. FOIAŞ, *Theory of Generalized Spectral Operators*, Gordon and Breach, Science Publishers, Inc., 1968.

[12] A. DIJKSMA AND H. S. V. DE SNOO, *Symmetric and selfadjoint relations in Krein spaces I*, Oper. Theory: Adv. Appl. **24** (1987), 145–166.

[13] U. GÜNTHER, F. STEFANI, AND M. ZNOJIL, *MHD α^2-dynamo, squire equation and $\mathcal{PT}$-symmetric interpolation between square well and harmonic oscillator*, J. Math. Phys. **46** (2005), 063504.

Paper OaM-09-30

[14] B. JACOB AND C. TRUNK, *Location of the spectrum of operator matrices which are associated to second order equations*, Operators and Matrices **1** (2007), 45–60.

[15] B. JACOB, C. TRUNK, AND M. WINKLMEIER, *Analyticity and Riesz basis property of semigroups associated to damped vibrations*, Journal of Evolution Equations **8** (2008), 263–281.

[16] P. JONAS AND H. LANGER, *Compact perturbations of definitizable operators*, J. Operator Theory **2** (1979), 63–77.

[17] T. KATO, *Perturbation Theory for Linear Operators*, Second Edition, Springer, 1976.

[18] M. KALTENBÄCK, H. WINKLER, AND H. WORACEK, *Almost Pontryagin spaces*, Oper. Theory Adv. Appl. **160** (2005), 253–271.

[19] G. KÖTHE, *Topological Vector Spaces I*, Springer 1969.

[20] H. LANGER, *Spectral functions of definitizable operators in Krein spaces*, in: Functional Analysis: Proceedings of a Conference Held at Dubrovnik, Yugoslavia, November 2–14, 1981; Lecture Notes in Mathematics **948** (1982), 1–46, Springer.

[21] H. LANGER, A. MARKUS, AND V. MATSAEV, *Locally definite operators in indefinite inner product spaces*, Math. Ann. **308** (1997), 405–424.

[22] H. LANGER AND C. TRETTER, *A Krein space approach to $\mathscr{P}\mathscr{T}$-symmetry*, Czech. J. Phys. **54** (2004), 1113–1120.

[23] I. LYUBICH AND V. I. MATSAEV, *On operators with decomposable spectrum*, Mat. Sbornik **56** (98) (1962), 433–468 (Russian). Engl. transl.: AMS Transl. (2) **47** (1965), 89–129.

[24] V. MÜLLER, *Spectral Theory of Linear Operators and Spectral Systems in Banach Algebras*, 2nd edition, Birkhäuser 2007.

[25] F. PHILIPP, *Indefinite Sturm-Liouville operators with periodic coefficients*, Oper. Matrices **7** (2013) 777–811.

[26] F. PHILIPP, V. STRAUSS, AND C. TRUNK, *Local spectral theory for normal operators in Krein spaces*, Math. Nachr. **286** (2013), 32–58.

[27] F. PHILIPP AND C. TRUNK, *G-selfadjoint operators in Almost Pontryagin spaces*, Oper. Theory: Adv. Appl. **188** (2008), 207–235.