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A GENERAL TOOL FOR DETERMINING THE ASYMPTOTIC SPECTRAL DISTRIBUTION OF HERMITIAN MATRIX–SEQUENCES

CARLO GARONI, STEFANO SERRA-CAPIZZANO AND PARIS VASSALOS

Abstract. We consider sequences of Hermitian matrices with increasing dimension, and we provide a general tool for determining the asymptotic spectral distribution of a 'difficult' sequence $\{A_n\}_n$ from the one of 'simpler' sequences $\{B_{n,m}\}_n$ that approximate $\{A_n\}_n$ when $m \to \infty$. The tool is based on the notion of an approximating class of sequences (a.c.s.), which was inspired by the work of Paolo Tilli and the second author, and it is applied here in a more general setting. An a.c.s.-based proof of the famous Szegö theorem on the spectral distribution of Toeplitz matrices is finally presented.

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