

FUNCTIONAL DECOMPOSITION THEOREMS FOR C^* -MATRIX OPERATOR SPACES

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Abstract. Let S be a nonempty set; and let \mathcal{A} be a fixed C^* -algebra with state space $s(\mathcal{A})$ equipped with the relative weak* topology inherited from the dual space $\mathcal{A}^\#$ of \mathcal{A} . Let \mathcal{X} be the space of all functions $\mathbf{x} : S \rightarrow \mathcal{A}$ such that $\varphi \circ (\mathbf{x}^* \mathbf{x}) \in \ell^1(S)$ for all $\varphi \in s(\mathcal{A})$, and the map $\varphi \rightarrow \varphi \circ (\mathbf{x}^* \mathbf{x})$ is weak* to norm continuous from $s(\mathcal{A})$ to $\ell^1(S)$. Elementary methods are used to show that the space \mathcal{M} of \mathcal{A} -valued functions on $S \times S$ that define bounded operators on \mathcal{X} contains a closed subspace \mathcal{K} such that the annihilator \mathcal{K}^\perp is an ℓ^1 direct summand of the dual space $\mathcal{M}^\#$ of \mathcal{M} ; i.e., \mathcal{M} contains an M -ideal. When \mathcal{A} is specialized to the complex field, this is a classical theorem of Dixmier. An analogue of the trace formula $\text{trace}(AB) = \text{trace}(BA)$ for a trace class operator A and a bounded operator B on a Hilbert space is proved.

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REFERENCES

- [1] E. M. ALFSEN AND E. G. EFFROS, *Structure in real Banach spaces*, Ann. of Math. **96** (1972), 98–173.
- [2] J. DIXMIER, *Les fonctionnelles linéaires sur l'ensemble des opérateurs bornés dans espace de Hilbert*, Ann. Math. **51** (1950), 387–408.
- [3] PETER HARMAND, DIRK WERNER, AND WEND WERNER, *M-Ideals in Banach Spaces and Banach Algebras*, Lecture Notes in Mathematics, vol. 1547, Springer-Verlag, 1993.
- [4] J. HENNEFELD, *A decomposition for $B(X)^*$ and unique Hahn-Banach extensions*, Pac. J Math. **46** (1973), 197–199.
- [5] R. V. KADISON AND J. R. RINGROSE, *Fundamentals of the Theory of Operator Algebras*, vol. I, Academic Press, New York, 1983.
- [6] E. C. LANCE, *Hilbert C^* -Modules – A toolkit for operator algebraists*, LMS Lecture Note Series, no. 210, Cambridge University Press, 1995.
- [7] R. R. SMITH AND J. D. WARD, *M-ideal structure in Banach algebras*, J. Funct. Anal. **27** (1978), 337–349.
- [8] T. WOITIJIRATTIKAL AND S.-C. ONG, *Functional decomposition of state induced C^* -matrix spaces*, Banach J. Math. Anal. **5** (2011), no. 2, 106–121.
- [9] T. WOITIJIRATTIKAL, S.-C. ONG, AND J. RAKBUD, *Functional decompositions on vector-valued function spaces via operators*, J. Math. Anal. Appl. **389** (2012), 1173–1190.
- [10] O. WOITIJIRATTIKAL, S.-C. ONG, P. CHAISURIYA, AND J. RAKBUD, *Banach spaces of functions taking values in a C^* -algebra*, Oper. Matrices **3** (2009), 373–396.