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# CLOSEST SOUTHEAST SUBMATRIX THAT MAKES MULTIPLE A DEFECTIVE EIGENVALUE OF THE NORTHWEST ONE

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*Abstract.* Given three complex matrices $A \in \mathbb{C}^{n\times n}, B \in \mathbb{C}^{n\times m}$ and $C \in \mathbb{C}^{m\times n}$, and given a defective eigenvalue $z_0$ of $A$, we study when the set $\mathcal{S}$ of matrices $X \in \mathbb{C}^{m\times m}$ such that $z_0$ is a multiple eigenvalue of the matrix

$$\begin{pmatrix} A & B \\ C & X \end{pmatrix}.$$

is nonempty. Moreover, when $\mathcal{S} \neq \emptyset$, given a fourth matrix $D \in \mathbb{C}^{m\times m}$ we find a matrix $X_0 \in \mathcal{S}$ such that

$$\|X_0 - D\| = \min\{\|X - D\| : X \in \mathcal{S}\}.$$

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