

HAHN–BANACH TYPE EXTENSION THEOREMS ON p -OPERATOR SPACES

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Abstract. Let $V \subseteq W$ be two operator spaces. Arveson-Wittstock-Hahn-Banach theorem asserts that every completely contractive map $\varphi : V \rightarrow \mathcal{B}(H)$ has a completely contractive extension $\tilde{\varphi} : W \rightarrow \mathcal{B}(H)$, where $\mathcal{B}(H)$ denotes the space of all bounded operators from a Hilbert space H to itself. In this paper, we show that this is not in general true for p -operator spaces, that is, we show that there are p -operator spaces $V \subseteq W$, an SQ_p space E , and a p -completely contractive map $\varphi : V \rightarrow \mathcal{B}(E)$ such that φ does not extend to a p -completely contractive map on W . Restricting E to L_p spaces, we also consider a condition on W under which every completely contractive map $\varphi : V \rightarrow \mathcal{B}(L_p(\mu))$ has a completely contractive extension $\tilde{\varphi} : W \rightarrow \mathcal{B}(L_p(\mu))$.

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