MAPS ON OPERATORS STRONGLY PRESERVING SHARP ORDER

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Abstract. We discuss the structure of additive transformations on $B(H)$ that are strongly monotone with respect to the $\overset{\sharp}{\leqslant}$-order and characterize them under the assumption of bijectivity. We also characterize bijective transformations on the sets of linear combinations of idempotents in $B(H)$ that are strongly monotone with respect to the $\overset{\sharp}{\leqslant}$-order.

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REFERENCES

[1] A. ALIEVA, A. GUTERMAN, *Monotone linear transformations on matrices are invertible*, Comm. in Algebra, **33** (2005), 3335–3352.

[2] A. BEN-ISRAEL, T. GREVILLE, *Generalized Inverses: Theory and Applications*, New York: Hohn Wiley and Sons. 1974.

[3] I. I. BOGDANOV, A. E. GUTERMAN, *Monotone matrix maps defined by the group inverse and simultaneous diagonalizability*, Matematicheskii Sbornik, **198**, 1 (2007), 3–20 [in Russian].

[4] M. BOHATA, J. HAMHALTER, *Nonlinear maps on von Neumann algebras preserving the star order*, Linear and Mult. Algebra, **61**, 7 (2013), 998–1009.

[5] M. BOHATA, J. HAMHALTER, *Star order on JBW algebras*, J. Math. Anal. Appl. **417**, 2 (2014), 873–888.

[6] CH. DENG, SH. WANG, *On some characterizations of the partial orderings for bounded operators*, Math. Inequal. Appl. **15**, 3 (2012), 619–630.

[7] G. DOLINAR, A. GUTERMAN, J. MAROVT, *Automorphisms of $K(H)$ with respect to the star partial order*, Operators and Matrices, **7**, 1 (2013), 225–239.

[8] G. DOLINAR, A. GUTERMAN, J. MAROVT, *Monotone transformations on $B(H)$ with respect to the left-star and the right-star partial order*, Mathematical Inequalities and Applications, **17**, 2 (2014), 573–589.

[9] G. DOLINAR, J. MAROVT, *Star partial order on $B(H)$*, Linear Algebra Appl., **434** (2011), 319–326.

[10] M. A. EFIMOV, *Additive matrix maps that are monotone with respect to the orders induced by group inverse*, Fundamentalnaya i Prikladnaya Matematika, **17**, 6 (2011/2012), 23–40 [in Russian]. Translated in Journal of Mathematical Sciences (New-York), **193**, 5 (2013), 659–670.

[11] M. A. EFIMOV, *Linear matrix transformations that are monotone with respect to the $\overset{\sharp}{\leqslant}$- or $\overset{cn}{\leqslant}$-order*, Fundamentalnaya i Prikladnaya Matematika, **13**, 4 (2007), 53–66 [in Russian]. Translated in Journal of Mathematical Sciences (New-York), **155**, 6 (2008), 830–838.

[12] M. A. EFIMOV, *On the $\overset{\sharp}{\leqslant}$-order on the set of linear bounded operators in Banach space*, Matematicheskie Zametki, **93**, 5 (2013), 794–797 [in Russian]. Translated in Mathematical Notes, **93**, 5 (2013), 784–788.

[13] M. A. EFIMOV, A. E. GUTERMAN, *Monotone maps on diagonalizable matrices*, Mathematical Inequalities and Applications, **17**, 4, (2014) 1441–1452.

[14] M. A. EFIMOV, A. E. GUTERMAN, *Monotone maps on index one matrices*, Zapiski POMI, **405** (2012), 67–96 [in Russian]. Translated in Journal of Mathematical Sciences (New-York), **191**, 1 (2013), 36–51.

[15] J. HAMHALTER, *Isomorphisms of ordered structures of abelian C_*-subalgebras of C_*-algebras*, J. Math. Anal. Appl. **383**, 2 (2011), 391–399.

[16] J. HAMHALTER, E. TURILOVA, *Automorphisms of order structures of abelian parts of operator algebras and their role in quantum theory*, Internat. J. Theoret. Phys. **53**, 10 (2014), 3333–3345.

[17] R. E. HARTWIG, *How to partially order regular elements*, Math. Japonica, **25**, 1 (1980), 1–13.

[18] R. E. HARTWIG, S. K. MITRA, *Partial orders based on outer inverses*, Linear Algebra Appl., **176** (1982), 3–20.

[19] P. LEGIŠA, *Automorphisms of M_n, partially ordered by rank subtractivity ordering*, Linear Algebra Appl., **389** (2004), 147–158.

[20] P. LEGIŠA, *Automorphisms of M_n, partially ordered by the star order*, Linear and Mult. Algebra, **54**, 3 (2006), 157–188.

[21] J. MAROVT, D. S. RAKIĆ, D. S. DJORDJEVIĆ, *Star, left-star, and right-star partial orders in Rickart $*$-rings*, Linear and Mult. Algebra, **63**, 2 (2015), 343–365.

[22] S. K. MITRA, *On group inverses and the sharp order*, Linear Algebra Appl., **92** (1987), 17–37.

[23] S. K. MITRA, P. BHIMASANKARAM, S. B. MALIK, *Matrix Partial Orders, Shorted Operators and Applications*, World Scientific Publishing Company. 2010.

[24] M. Z. NASHED (Ed.), *Generalized inverses and applications*, Academic Press, New York-London, 1976.

[25] P. G. OVCHINNIKOV, *Automorphisms of the poset of skew projections*, J. of Functional Analysis, **115** (1993), 184–189.

[26] J. DE PILLIS, *Linear transformations which preserve Hermitian and positive semidefinite operators*, Pacific J. of Math., **23** (1967), 129–137.

[27] D. S. RAKIĆ, D. S. DJORDJEVIĆ, *Space pre-order and minus partial order for operators on Banach spaces*, Aequationes Math. **85**, 3 (2013), 429–448.

[28] P. ŠEMRL, *Order-preserving maps on the poset of idempotent matrices*, Acta Sci. Math. (Szeged), **63** (2003), 481–490.

[29] P. ŠEMRL, *Automorphisms of B(H) with respect to minus partial order*, J. Math. Anal. Appl, **369** (2010), 205–213.

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