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ON SPACES DERIVABLE FROM A SOLID SEQUENCE SPACE AND A NON–NEGATIVE LOWER TRIANGULAR MATRIX

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Abstract. The scalar field will be either the real or complex numbers. Suppose that λ is a solid sequence space over the scalar field and A is an infinite lower triangular matrix with non-negative entries and positive entries on the main diagonal such that each of its columns is in λ. For each positive integer k, the k^{th} predecessor of λ with respect to A is the solid vector space of scalar sequences x such that $A^k|x|$ is an element of λ. We denote this space by Λ_k and λ itself will be denoted by Λ_0. Under reasonable assumptions, these spaces inherit some topological properties from λ. We are interested in a projective limit of the infinite product of the Λ_k consisting of sequences of sequences $(x^{(k)})$ satisfying $Ax^{(k)} = x^{(k-1)}$ for each $k > 0$. We show that for interesting classes of situations including the cases when $\lambda = l_p$ for some $p > 1$ and A is the Cesàro matrix, the space of our interest can be non-trivial.

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