A SHORT–TYPE DECOMPOSITION OF FORMS

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Abstract. The main purpose of this paper is to present a decomposition theorem for nonnegative sesquilinear forms. The key notion is the short of a form to a linear subspace. This is a generalization of the well-known operator short defined by M. G. Krein. A decomposition of a form into a shorted part and a singular part (with respect to an other form) will be called short-type decomposition. As applications, we present some analogous results for bounded positive operators acting on a Hilbert space; for additive set functions on a ring of sets; and for representable positive functionals on a $*$-algebra.

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