DENSE DEFINITENESS AND BOUNDEDNESS OF COMPOSITION OPERATORS IN L^2–SPACES VIA INDUCTIVE LIMITS

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Abstract. The questions of dense definiteness and boundedness of composition operators in L^2-spaces are studied by means of inductive limits of operators. Methods based on projective systems of measure spaces and inductive limits of L^2-spaces are developed. Illustrative examples are presented.

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