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ASYMPTOTIC BEHAVIOR OF EIGENVALUES AND EIGENFUNCTIONS OF STURM–LIOUVILLE PROBLEMS WITH COUPLED BOUNDARY CONDITIONS AND TRANSMISSION CONDITIONS

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Abstract. The Sturm-Liouville (S-L) problems with coupled boundary conditions and transmission conditions are investigated. By defining a new Hilbert space which is related to the transmission conditions, the self-adjointness of the S-L problems in this associated Hilbert space is proved, and the asymptotic behavior of eigenvalues and eigenfunctions of the problem are described. We also give the condition for λ being the eigenvalue of the S-L problems with coupled boundary conditions.

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