Operators and Matrices
Volume 9, Number 4 (2015), 891–905
doi:10.7153/oam-09-52

ONE–SIDED STAR PARTIAL ORDERS FOR BOUNDED LINEAR OPERATORS

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Abstract. We compare some recent approaches to transferring the notions of left- and right-star partial order, introduced for complex matrices in early 90-ies, to bounded linear Hilbert space operators, and discuss a new version of these orders. The main results state that every initial segment of $\mathscr{B}(H)$ under the (new) left-star order is a complete orthomodular sublattice isomorphic to an initial segment of the lattice of closed subspaces of the underlying Hilbert space H. We also associate a certain orthogonality relation with the order.

The so called logical order on the set of all self-adjoint operators, introduced by S. Gudder in 2006, turns out to be the restriction of any of both one-sided star orders. Various known results concerning the logical order, in particular, characterizations of the join and meet operations, are extended to the left-star order on $\mathscr{B}(H)$.

Mathematics subject classification (2010): 06A06, 06C15, 46A32, 46C05, 47A05.

Keywords and phrases: bounded linear operator; left-star order; left-star orthogonality; orthogonal projection operator; orthomodular lattice; right-star order; right-star orthogonality.

REFERENCES

[1] J. ANTEZANA, C. CANO e.a., *A note on the star order in Hilbert spaces*, Linear Multilinear Algebra **58** (2010), 1037–1051, DOI:10.1080/03081080903227104 .

[2] J.K. BAKSALARY, S.K. MITRA, *Left-star and right-star partial ordering*, Linear Algebra Appl. **149** (1991), 73–89.

[3] G. CHEVALIER, *Order and orthogonality relations in rings and algebras*, Tatra Mountains Math. Publ. **3** (1992), 31–46.

[4] J. CĪRULIS, *Further remarks on an order of quantum observables*, Math. Slovaca (in print; a preprint available as arXiv:1301.0640v2).

[5] J. CĪRULIS, *Lattice operators on Rickart *-rings under the star order*, Linear Multilinear Algebra **63** (2015), 497-508, DOI: 10.1080/03081087.2013.873429 .

[6] J. CĪRULIS, *On one-side star partial orders on a Rickart *-ring*, arXiv:1410.4693v1, 2014.

[7] CH. DENG, *Some properties on the star order of bounded operators*, J. Math. Anal. Appl. **423** (2014), 32–40, http.//dx.doi.org/10.1016/j.jmaa.2014.09.077 .

[8] CH. DENG, SH. WANG, *On some characterizations of the partial ordering for bounded operators*, Math. Inequal. Appl. **12** (2012), 619–630.

[9] G. DOLINAR, A. GUTERMAN, J. MAROVT, *Monotone transformations on B(H) with respect to the left-star and the right-star partial order*, Math. Inequal. Appl. **17** (2014), 573–589.

[10] G. DOLINAR, J. MAROVT, *Star partial order on B(H)*, Linear Algebra Appl. 434 (2011), 319–326.

[11] S. GUDDER, *An order for quantum observables*, Math. Slovaca **56** (2006), 573–589.

[12] J. MAROVT, D.S. RAKIĆ, D.S. DJORDJEVIĆ, *Star, left-star, and right-star partial orders in Rickart *-rings*, Linear Multilinear Algebra **63** (2015), 343-365, DOI: 10.1080/03081087.2013.866670 .

[13] S.K. MITRA, P. BHIMASANHARAM, S.B. MALIK, *Matrix Partial Orders, Shorted Operators and Applications*. Singapore: World Scientific, 2010.

[14] S. PULMANNOVÁ, E. VINCEKOVÁ, *Remarks on the order for quantum observables*, Math. Slovaca **57** (2007), 589–600.

Paper OaM-09-52