

SOME EXAMPLES OF EXTREMAL TRIPLES OF COMMUTING CONTRACTIONS

EDWARD J. TIMKO

Abstract. The collection $\mathfrak{E}_3$ of all triples of commuting contractions forms a family in the sense of Agler, and so has an “optimal” model $\partial\mathfrak{E}_3$ generated by its extremal elements. A given $T \in \mathfrak{E}_3$ is extremal if every $X \in \mathfrak{E}_3$ extending T is an extension by direct sum. We show that many of the known examples of triples in $\mathfrak{E}_3$ that fail to have coisometric extensions are in fact extremal.

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