

INEQUALITIES RELATED TO 2×2 BLOCK PPT MATRICES

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Abstract. A 2×2 block matrix $\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}$ is positive partial transpose (PPT) if both $\begin{bmatrix} A & X \\ X^* & B \end{bmatrix}$ and $\begin{bmatrix} A & X^* \\ X & B \end{bmatrix}$ are positive semidefinite. This article presents some inequalities related to this class of matrices. Among others, we show that the Hua matrix, which is PPT, reveals a remarkable singular value inequality for contractive matrices.

Mathematics subject classification (2010): 15A45, 15A42, 47A30.

Keywords and phrases: Positive partial transpose, inequality, trace, singular value.

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