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IDEAL–TRIANGULARIZABILITY AND COMMUTATORS OF CONSTANT SIGN

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Abstract. Let E be a Banach lattice with order continuous norm, and let A and B be positive compact operators such that the commutator $AB-BA$ is also positive. We prove that if A and B are ideal-triangularizable, then they are simultaneously ideal-triangularizable, or equivalently, the sum $A+B$ is ideal-triangularizable. We then show several related results for operators of constant sign (an operator T on E is of constant sign if either T or $-T$ is positive). In particular, we consider ideal-triangularizability for Lie sets of compact operators of constant sign (a set of operators is a Lie set whenever it is closed under taking commutators).

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