

## A SUBNORMAL TOEPLITZ COMPLETION

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*Abstract.* In this paper we deal with a subnormal Toeplitz completion problem: Complete the unspecified Toeplitz operators of the partial block Toeplitz matrix

$$G := \begin{bmatrix} U^{*p} & ? \\ ? & U^{*q} \end{bmatrix} \quad (p, q = 1, 2, \dots)$$

to make  $G$  subnormal, where  $U$  is the shift on the Hardy space  $H^2(\mathbb{T})$  of the unit circle  $\mathbb{T}$ .

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