

## PRESERVERS OF PSEUDO SPECTRA OF OPERATOR JORDAN TRIPLE PRODUCTS

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*Abstract.* Let  $\mathcal{H}$  be an infinite-dimensional complex Hilbert space and let  $\mathcal{L}(\mathcal{H})$  be the algebra of all bounded linear operators on  $\mathcal{H}$ . For  $\varepsilon > 0$  and  $T \in \mathcal{L}(\mathcal{H})$ , let  $r_\varepsilon(T)$  denote the  $\varepsilon$ -pseudo spectral radius of  $T$ . We characterize surjective maps  $\phi$  on  $\mathcal{L}(\mathcal{H})$  which satisfy

$$r_\varepsilon(\phi(T)\phi(S)\phi(T)) = r_\varepsilon(TST)$$

for all  $T, S \in \mathcal{L}(\mathcal{H})$ . As application, mappings from  $\mathcal{L}(X)$  onto itself that preserve the pseudo spectrum of Jordan triple product of operators are described. We also obtain analogous results for the finite-dimensional case, without the surjectivity assumption on  $\phi$ .

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### REFERENCES

- [1] M. BENDAOU, *Preservers of local spectrum of matrix Jordan triple products*, Linear Algebra Appl. **471**, 1 (2015), 604–614.
- [2] M. BENDAOU, *Preservers of local spectra of matrix sums*, Linear Algebra Appl. **438**, 5 (2013), 2500–2507.
- [3] M. BENDAOU, M. DOUMI AND M. SARIH, *Maps on matrices preserving local spectra*, Linear Multilinear Algebra **61**, 7 (2013), 871–880.
- [4] M. BENDAOU, M. JABBAR AND M. SARIH, *Preservers of local spectra of operator products*, Linear Multilinear Algebra **63**, 4 (2015), 806–819.
- [5] R. BHATIA, P. ŠEMRL AND A. R. SOUROUR, *Maps on matrices that preserve the spectral radius distance*, Studia Math. **134**, 1–3 (1999), 99–110.
- [6] J. T. CHAN, C. K. LI AND N. S. SZE, *Mappings preserving spectra of products of matrices*, Proc. Amer. Math. Soc. **135**, 4 (2007), 977–986.
- [7] J. T. CHAN, C. K. LI AND N. S. SZE, *Mappings on matrices: invariance of functional values of matrix products*, J. Austral. Math. Soc. **81**, 2 (2006), 165–184.
- [8] J. CUI, V. FORSTALL, C.-K. LI AND V. YANNELLO, *Properties and preservers of the pseudospectrum*, Linear Algebra Appl. **436**, 2 (2012), 316–325.
- [9] J. CUI AND J. HOU, *Maps leaving functional values of operator products invariant*, Linear Algebra Appl. **428**, 7 (2008), 1649–1663.
- [10] J. CUI AND C.-K. LI, *Maps preserving peripheral spectrum of Jordan products of operators*, Oper. Matrices **6**, 6 (2012), 129–146.
- [11] J. CUI, C. K. LI AND Y. T. POON, *Pseudospectra of special operators and pseudospectrum preservers*, J. Math. Anal. Appl. **419**, 2 (2014), 1261–1273.
- [12] J. CUI, C. K. LI AND Y. T. POON, *Preservers of unitary similarity functions on Lie products of matrices*, Linear Algebra Appl. (2015), doi:10.1016/j.laa.2015.02.036.
- [13] G. DOBOVIŠEK, B. KUZMA, G. LEŠNJAK, C. K. LI AND T. PETEK, *Mappings that preserve pairs of operators with zero triple Jordan product*, Linear Algebra Appl. **426**, 2–3 (2007), 255–279.

- [14] G. DOLINAR, J. HOU, B. KUZMA AND X. QI, *Spectrum nonincreasing maps on matrices*, Linear Algebra Appl. **438**, 8 (2013), 3504–3510.
- [15] H. GAO, *\*-Jordan-triple multiplicative surjective maps on  $B(H)$* , J. Math. Anal. Appl. **401**, 1 (2013), 397–403.
- [16] J. C. HOU, C. K. LI AND N. C. WONG, *Maps preserving the spectrum of generalized Jordan product of operators*, Linear Algebra Appl. **432**, 4 (2010), 1049–1069.
- [17] G. K. KUMAR AND S. H. KULKARNI, *Linear maps preserving pseudospectrum and condition spectrum*, Banach J. Math. Anal. **6**, 1 (2012), 45–60.
- [18] L. MOLNÁR, *Some characterizations of the automorphisms of  $B(H)$  and  $C(H)$* , Proc. Amer. Math. Soc. **130**, 1 (2001), 111–120.
- [19] L. N. TREFETHEN AND M. EMBREE, *Spectra and Pseudospectra, The Behavior of Nonnormal Matrices and Operators*, Princeton University Press, Princeton, 2005.
- [20] P. B. YALE, *Automorphism of the complex numbers*, Math. Mag. **39**, 2 (1966), 135–141.
- [21] W. ZHANG AND J. HOU, *Maps preserving peripheral spectrum of Jordan semi-triple products of operators*, Linear Algebra Appl. **435**, 6 (2011), 1326–1335.