

GENERIC RANK-TWO PERTURBATIONS OF STRUCTURED REGULAR MATRIX PENCILS

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Abstract. The generic spectral behavior of classes of structured regular matrix pencils is examined under structure-preserving rank-2 perturbations, i.e., perturbations of normal rank two. For T -alternating, palindromic, and skew-symmetric matrix pencils we observe the following effects at each eigenvalue λ under a generic, structure-preserving rank-2 perturbation: 1) The largest two Jordan blocks at λ are destroyed. 2) If hereby the eigenvalue pairing imposed by the structure is violated, also the largest remaining Jordan block at λ will grow in size by one. 3) If λ is a single (double) eigenvalue of the perturbing pencil, one (two) new Jordan blocks of size one will be created at λ .

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