POSITIVE DEFINITE SOLUTIONS OF
CERTAIN NONLINEAR MATRIX EQUATIONS

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Abstract. We investigate positive definite solutions of nonlinear matrix equations $X - f(\Phi(X)) = Q$ and $X - \sum_{i=1}^{m} f(\Phi_i(X)) = Q$, where Q is a positive definite matrix, Φ and Φ_i $(1 \leqslant i \leqslant m)$ are positive linear maps on $\mathbb{M}_n(\mathbb{C})$ and f is a nonnegative matrix monotone or matrix anti-monotone function on $[0, \infty)$. In this article, using appropriate inequalities and some fixed point results, we prove the existence of unique positive definite solutions for the mentioned above equations.

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