

## PERIPHERAL LOCAL SPECTRUM PRESERVERS AND MAPS INCREASING THE LOCAL SPECTRAL RADIUS

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*Abstract.* We address two long standing problems in the context of local spectral radius preservers. First, we completely describe the form of maps preserving the peripheral local spectrum of product or triple product of operators. Second, we establish the automatic continuity of linear maps increasing the local spectral radius of operators at a fixed nonzero vector.

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