

ON THE SUMMATION OF FRACTIONAL POWERS OF MATRICES OVER FINITE FIELDS

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Abstract. In this paper, we investigate the summation of fractional powers of matrices over finite fields. More specifically, we show how a fractional power of a matrix of a linear transform over a finite field can be obtained from the linear combination of some specific powers of the same matrix. We use the developed theory to construct matrices related to fractional Fourier and cosine transforms over finite fields.

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