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ON THE HOMOGENIZATION OF PARTIAL INTEGRO–DIFFERENTIAL–ALGEBRAIC EQUATIONS

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Abstract. We present a Hilbert space perspective to homogenization of standard linear evolutionary boundary value problems in mathematical physics and provide a unified treatment for (non-)periodic homogenization problems in thermodynamics, elasticity, electro-magnetism and coupled systems thereof. The approach permits the consideration of memory problems as well as differential-algebraic equations. We show that the limit equation is well-posed and causal. We rely on techniques from functional analysis and operator theory only.

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