

## THE STRUCTURE OF $m$ -ISOMETRIC WEIGHTED SHIFT OPERATORS

BELAL ABDULLAH AND TRIEU LE

*Abstract.* We obtain simple characterizations of unilateral and bilateral weighted shift operators that are  $m$ -isometric. We show that any such operator is a Hadamard product of 2-isometries and 3-isometries. We also study weighted shift operators whose powers are  $m$ -isometric.

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