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NONLINEAR MAPS PRESERVING HIGHER–DIMENSIONAL NUMERICAL RANGE OF SKEW LIE PRODUCT OF OPERATORS

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Abstract. Let k be a positive integer. Let H and K be complex Hilbert spaces of dimensions greater than k. By $W_k(A)$ denote the k-dimensional numerical range of an operator A. In this paper we prove that a surjective map $\phi : B(H) \to B(K)$ satisfies $W_k(AB - BA^*) = W_k(\phi(A)\phi(B) - \phi(B)\phi(A)^*)$ for all $A, B \in B(H)$ if and only if there exists a unitary operator $U \in B(H,K)$ such that $\phi(A) = \gamma UAU^*$ for all $A \in B(H)$, where $\gamma \in \{-1,1\}$.

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