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MULTIPLICATIVE LIE HIGHER DERIVATIONS OF UNITAL ALGEBRAS WITH IDEMPOTENTS

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Abstract. Let $\mathscr{R}$ be a commutative ring with identity and $\mathscr{A}$ be a unital algebra with nontrivial idempotent e over $\mathscr{R}$. Motivated by Benkovič's systematic and powerful work [2, 3, 4, 5, 6, 7, 8], we will study multiplicative Lie higher derivations (i.e. those Lie higher derivations without additivity assumption) on $\mathscr{A}$ in this article. Let $D = \{L_k\}_{k\in\mathbb{N}}$ be a multiplicative Lie higher derivation on $\mathscr{A}$. It is shown that under suitable assumptions, $D = \{L_k\}_{k\in\mathbb{N}}$ is of standard form; i.e. each component L_k $(k \geqslant 1)$ can be expressed through an additive higher derivation and a central mapping vanishing on all commutators of $\mathscr{A}$.

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