ADDITIVE MAPS PRESERVING $m$–NORMAL EIGENVALUES ON $\mathcal{B}(\mathcal{H})$

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Abstract. Let $\mathcal{H}$ be an infinite-dimensional complex Hilbert space and $\mathcal{B}(\mathcal{H})$ the algebra of all bounded linear operators on $\mathcal{H}$. For an operator $T \in \mathcal{B}(\mathcal{H})$ and a fixed non-negative integer $m$, an $m$-normal eigenvalue $\lambda$ of $T$ is the normal eigenvalue satisfying $\dim N(T - \lambda I) > m$. In this paper, we prove that, if an additive surjective map $\varphi$ on $\mathcal{B}(\mathcal{H})$ preserves $m$ as well as $m + 1$-normal eigenvalues, then there is an invertible operator $A \in \mathcal{B}(\mathcal{H})$ such that $\varphi(T) = ATA^{-1}$ for all $T \in \mathcal{B}(\mathcal{H})$ or $\varphi(T) = AT^\text{tr}A^{-1}$ for all $T \in \mathcal{B}(\mathcal{H})$, where $T^\text{tr}$ denotes the transpose of $T$ with respect to an arbitrary but fixed orthonormal basis of $\mathcal{H}$.


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REFERENCES