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ADDITIVE MAPS PRESERVING m–NORMAL EIGENVALUES ON $\mathscr{B}(\mathscr{H})$

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Abstract. Let $\mathscr{H}$ be an infinite-dimensional complex Hilbert space and $\mathscr{B}(\mathscr{H})$ the algebra of all bounded linear operators on $\mathscr{H}$. For an operator $T \in \mathscr{B}(\mathscr{H})$ and a fixed non-negative integer m, an m-normal eigenvalue λ of T is the normal eigenvalue satisfying $\dim N(T-\lambda I) > m$. In this paper, we prove that, if an additive surjective map φ on $\mathscr{B}(\mathscr{H})$ preserves m as well as $m+1$-normal eigenvalues, then there is an invertible operator $A \in \mathscr{B}(\mathscr{H})$ such that $\varphi(T) = ATA^{-1}$ for all $T \in \mathscr{B}(\mathscr{H})$ or $\varphi(T) = AT^{tr}A^{-1}$ for all $T \in \mathscr{B}(\mathscr{H})$, where T^{tr} denotes the transpose of T with respect to an arbitrary but fixed orthonormal basis of $\mathscr{H}$.

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REFERENCES

[1] B. AUPETIT, *Spectrum-preserving linear mapping between Banach algebra or Jordan Banach algebras*, J. London Math. Soc. **62**, (2000), 917–924.
[2] M. BENDAOUD, A. BOURHIM, M. BURGOS AND M. SARIH, *Linear maps preservig Fredholm and Atkinson elements of C^*-algebra*, Linear Multilinear Alg. **57**, (2009), 823–838.
[3] X. CAO, S. CHEN, *Linear maps between operator algebras preserving certain spectral functions*, Banach J. Math. Anal. **8**, (2014), 39–46.
[4] J. CUI AND J. HOU, *Additive maps on standard operator algebras preserving parts of the spectrum*, J. Math. Anal. Appl. **282**, (2003), 266–278.
[5] J. DIEUDONNÉ, *Sur une généralisation du groupe orthogonal à quatre variables*, Arch. Math. (Basel) **1**, (1949), 282–287.
[6] G. JI, Y. GAO, *Maps preserving operator pairs whose products are projections*, Linear Algebra Appl. **433**, (2010), 1348–1364.
[7] L. LIU, G. JI, *Maps preserving product X^*Y+YX^* on factor von Neumann algebras*, Linear and Multilinear Algebra. **59**, (2011), 950–955.
[8] D. A. HERRERO, *Approximation of Hilbert space operators*, Vol. 1, 2nd ed., Pitman Res. Notes Math. Ser., Vol. 224, Longman Scientific and Technical, Harlow, 1989.
[9] J. HOU, *Spectrum-preserving elementary operators on $B(X)$*, Chinese Annals of Mathematics. **19**, (1998), 511–516.
[10] A. A. JAFARIAN AND A. R. SOUROUR, *Spectrum-preserving linear maps*, J. Funct. Anal. **66**, (1986), 255–261.
[11] M. MARCUS AND R. PURVES, *Linear transformations on algebras of matrices: The invariance of the elementary symmetric functions*, Canad. J. Math. **11**, (1959), 383–396.
[12] M. MBEKHTA, V. MÜLLER, M. OUDGHIRI, *Additive preservers of the ascent, descent and related subsets*, to appear in journal of Operator Theory, 2012.
[13] M. OMLADI, P. ŠEMRL, *Additive mappings preserving operators of rank one*, Linear Alg. Appl. **182**, (1993), 239–256.

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