

## SINGULAR VALUE INEQUALITIES RELATED TO THE AUDENAERT–ZHAN INEQUALITY

HONGLIANG ZUO, MASATOSHI FUJII, JUNICHI FUJII AND YUKI SEO

*Abstract.* In this paper, we refine the Heinz mean inequality for singular values and give some generalizations of Audenaert-Zhan inequality for singular values and Zhan's conjecture for the case of negative  $t$ . Among others, we show that if  $A, B \in \mathbb{M}_n$  are positive semidefinite and  $f, g$  are real valued continuous functions on  $[0, \infty)$  such that  $g$  is monotone and  $f(g^{-1}(\sqrt{t}))^2$  is operator monotone on  $[0, \infty)$ , then

$$s_j(f(A)(g(A)^2 + g(B)^2)f(B)) \leq \frac{1}{2}s_j(f(A)^2g(A)^2 + f(B)^2g(B)^2)$$

for  $j = 1, \dots, n$ , where  $s_j$  are the singular values in decreasing order.

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