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NOTE ON SOME OPERATOR EQUATIONS AND LOCAL SPECTRAL PROPERTIES

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Abstract. In this paper we define $\mathscr{S}_{k,j}$ by the set of solutions (A,B) of the operator equations $A^kB^{j+1}A^k = A^{2k+j}$ and $B^kA^{j+1}B^k = B^{2k+j}$. Then we observe the set $\mathscr{S}_{k,j}$ is increasing for all integers $k \geqslant 1$ and $j \geqslant 0$.

Now let a pair $(A,B) \in \mathscr{S}_{k,j} \cap \mathscr{S}_{j+1,k-1}$ for any integer $k \geqslant 1$ and $j \geqslant 0$. We show that if any one of the operators A, AB, BA, and B has Bishop's property (β), then all others have the same property. Furthermore, we prove that the operators A^{k+j}, A^kB^{j+1}, $A^{j+1}B^k$, $B^{j+1}A^k$, B^kA^{j+1} and B^{k+j} have the same spectra and spectral properties. Finally, we investigate their Weyl type theorems.

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