NOTE ON SOME OPERATOR EQUATIONS
AND LOCAL SPECTRAL PROPERTIES

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Abstract. In this paper we define $\mathcal{S}_{k,j}$ by the set of solutions $(A, B)$ of the operator equations $A^kB^{j+1}A^k = A^{2k+j}$ and $B^kA^{j+1}B^k = B^{2k+j}$. Then we observe the set $\mathcal{S}_{k,j}$ is increasing for all integers $k \geq 1$ and $j \geq 0$.

Now let a pair $(A, B) \in \mathcal{S}_{k,j} \cap \mathcal{S}_{j+1,k-1}$ for any integer $k \geq 1$ and $j \geq 0$. We show that if any one of the operators $A$, $AB$, $BA$, and $B$ has Bishop’s property $(\beta)$, then all others have the same property. Furthermore, we prove that the operators $A^{k+j}$, $A^kB^{j+1}$, $A^{j+1}B^k$, $B^{j+1}A^k$, $B^kA^{j+1}$ and $B^{k+j}$ have the same spectra and spectral properties. Finally, we investigate their Weyl type theorems.


Keywords and phrases: Operator equations, spectrum, single valued extension property.

REFERENCES